#### **REVIEW**

on the PhD dissertation of Zhami Bakytzhan entitled «Investigation of hot rotating white dwarfs in general relativity», submitted for the degree of Doctor of Philosophy (PhD) in the specialty «6D060400 – Physics»

## 1. The relevance of the research topic and its connection with general scientific and national programs.

The thesis is devoted to the theoretical research of properties of equilibrium configurations of white dwarfs taking into account the effects of general relativity, finite temperatures, nuclear composition and rotation.

A white dwarf is the most common end-product in the evolution of main sequence stars with low and medium masses. On the other hand, white dwarf is one of the classes of compact objects. A white dwarf is also called a degenerate dwarf by reason of it is mostly composed of electron-degenerate matter.

At the present, the construction of a realistic physical model of white dwarfs and development of existing models in accordance with observational data is one of actual problems of astrophysics. From this point of view, it is natural to expect in order to build a correct model of white dwarfs all physical effects which make significant contribution to the structure of white dwarfs must be properly taken into account. This indicates the relevance of this dissertation work.

Furthermore, white dwarfs are important to understand the expansion of the universe in cosmology in terms of type Ia supernova explosions, since they can provide independent information about the age of our galaxy and their distribution contains information about star formation history and subsequent evolution. On the hand, type Ia supernovae are used as standard candles in astronomy to estimate distances to remote galaxies and study the nature of dark energy. Since the progenitors of white dwarfs lose carbon, nitrogen and oxygen on the stage of the main sequence star, they make a significant contribution to the chemical evolution of our galaxy, and possibly are an important source of life sustaining chemical compounds.

In accordance with the foregoing, the relevance of the dissertation work is not in doubt and the theme of the work is related to both general scientific and national research programs.

#### 2. Scientific results and their validity.

The most significant scientific results of the dissertation formulated in the provisions are:

- 1. The mathematical and physical equivalence of the approximate stationary axially symmetric the Hartle-Thorne and Sedrakyan-Chubaryan solutions of Einstein field equations (up to second order in angular velocity) that describe the gravitational field of astrophysical compact objects including white dwarfs in the limiting case of slow rotation and small deformation.
- 2. The effects of general relativity become significant in the mass-radius relation for white dwarf masses closer to the Chandrasekhar limit, thermal effects

are most clearly pronounced for white dwarf masses lower than the Chandrasekhar limit, while rotation and nuclear composition are important for the entire mass range of white dwarfs.

- 3. The core temperatures of white dwarf-satellites in the binary systems of millisecond pulsars PSR J1738+0333, PSR J1012+5307 and PSR J1911-5958A estimated to be in the range  $(1.55-6.5)\times10^7$  K,  $(1.4-2.4)\times10^7$  K and  $(0.4-9.5)\times10^7$  K, respectively.
- 4. The I-Love-Q (the moment of inertia, the rotational Love number and the quadrupole moment) and I-Q-e (the moment of inertia, the quadrupole moment and the eccentricity) relations are universal and independent of the equations of state (Chandrasekhar and Salpeter equations of state) of white dwarfs.

### 3. The degree of validity and reliability of each scientific result (scientific provision), conclusions of the applicant, formulated in the thesis.

The results obtained in the dissertation are compared, where it is possible, with the works by other authors and with available observational data. The reliability of the scientific provisions submitted for defense, the results and conclusions of the applicant formulated in the dissertation is not in doubt. Moreover, the reliability and validity of the thesis results are confirmed by publications in the peer-reviewed foreign and local journals and approbations at international conferences. More precisely, according to materials presented in the dissertation 30 works have been published in total. 6 articles were published in the international journals and conference proceedings indexed in Scopus (Elsevier, Netherlands) and Web of Science (Clarivate Analytics, USA). 7 articles were published in journals recommended by the Committee for the Control of Education and Science of the Ministry of Education and Science of the Republic of Kazakhstan. 1 article was published in the scientific journal of the Republic of Kazakhstan. 16 abstracts and articles were published in the materials of local and foreign international scientific conferences.

# 4. The degree of novelty of each scientific result (scientific provision), conclusions of the applicant, formulated in the thesis.

The results obtained and presented in the dissertation of Zhami B.A. are new. In terms of the novelty of dissertation work, it can be concluded that the following results were obtained for the first time:

- A set of algebraic expressions relating the total mass, angular momentum and mass quadrupole moment of the Hartle-Thorne solution with the integration constants of the Sedrakyan-Chubaryan solution. Alternatively, the relevant multipole moments of both solutions have been calculated and it has been shown that they are identical. The mathematical and physical equivalence of the two metrics has also been proven.
- The mass-radius relations of white dwarfs taking into account the effects of general relativity, finite temperatures, nuclear composition and rotation were calculated.
- The core temperatures of white dwarf-satellites in the binary systems of millisecond pulsars PSR J1738+0333, PSR J1012+5307 and PSR J1911-5958A, have been estimated using the graphical method.

 The I-Love-Q and I-Love-e relations for white dwarfs have been shown to be universal and independent of the Chandrasekhar and Salpeter equations of state.

5. The practical and theoretical significance of scientific results.

The candidate has completed a piece of research that demonstrates a significant and original contribution to the knowledge in his field of study.

The results obtained in the work expand knowledge on the physics of white dwarfs and they are important from theoretical and practical point of view since they can be directly used in the further investigations of problems of physics of white dwarfs, relativistic astrophysics, cosmology, astronomy and in the observational data analyzes.

### 6. Remarks, suggestions on the dissertation.

The text of the dissertation contains minor grammatical and stylistic errors and typos, in particular, I list:

- 1. There should be "dash sign" instead of "hyphen sign" at the some references in the reference list, therefore, the design of the reference list should be brought into line with the generally accepted standard;
- 2. The equations 4.5-4.9, 4.10 should be written in the 14-th font, not in larger fonts;
  - 3. The name of subsection 4.1 should be started from a new page;
- 4. On the figures 5.4 and 5.5 the dimensions of the moment of inertia are indicated as  $g \times cm$  and  $g \times cm^3$ , respectively, but they should be  $g \times cm^2$ ;
- 5. It is necessary to keep the proportions of figures and axis labels, relative to the scale of the text and the page itself. For example, for the fig. 5.18 and fig. 5.20;
- 6. Necessary to write more clearly the difference between the Hartle-Thorne and Sedrakyan-Chybaryan metrics.

However, the above minor remarks do not have a principle character and do not reduce the scientific and practical value, and the positive assessment of Zhami Bakytzhan's thesis on the topic «Investigation of hot rotating white dwarfs in general relativity».

7. Compliance of the content of dissertation with the requirements of the «Rules for the award of scientific degrees».

Based on the foregoing, I believe that the dissertation work of Zhami B.A. on the topic «Investigation of hot rotating white dwarfs in general relativity», submitted for the degree of Doctor of Philosophy (PhD) in the specialty «6D060400 – Physics», meets the requirements for works of this kind, both in content and volume, and the candidate Zhami B.A. deserves the award of the degree of Doctor of Philosophy (PhD) in the specialty «6D060400 – Physics».

Official reviewer,

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